Question No: 1 (Marks: 1) - Please choose one
If for a linear transformation the equation $T(x) = 0$ has only the trivial solution then $T$ is

► one-to-one
► onto

Page no : 113

Question No: 2 (Marks: 1) - Please choose one
Which one of the following is an elementary matrix?

$\begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & -3 \end{bmatrix}$
$\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$
$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

can be converted into $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by $R2 + (-R1) \Rightarrow R2$

Ref: An elementary matrix is a matrix that results from applying a single elementary row operation to an identity matrix.

Question No: 3 (Marks: 1) - Please choose one
\[ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \]

Let and let \( k \) be a scalar. A formula that relates \( \det kA \) to \( k \) and \( \det A \) is

- \( \det kA = k \det A \)  
- \( \det kA = \det (k + A) \)  
- \( \det kA = k^2 \det A \)  
- \( \det kA = \det A \)

**Question No: 4  ( Marks: 1 ) - Please choose one**

The equation \( x = p + t \cdot v \) describes a line

- Through \( v \) parallel to \( p \)
- Through \( p \) parallel to \( v \)  
- Through origin parallel to \( p \)

**Question No: 5  ( Marks: 1 ) - Please choose one**

Determine which of the following sets of vectors are linearly dependent.

- \( v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \), \( v_2 = \begin{bmatrix} 6 \\ 2 \end{bmatrix} \)

- \( v_1 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \), \( v_2 = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix} \)

- \( v_1 = \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} \), \( v_2 = \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix} \)

**Question No: 6  ( Marks: 1 ) - Please choose one**

Every linear transformation is a matrix transformation
Question No: 7  ( Marks: 1 ) - Please choose one
A null space is a vector space.

► True  page no: 270
► False

Question No: 8  ( Marks: 1 ) - Please choose one
If two row interchanges are made in succession, then the new determinant

► equals to the old determinant

► equals to -1 times the old determinant  Page no : 386

Ref: A row interchange changes the sign of the determinant

Question No: 9  ( Marks: 1 ) - Please choose one
The determinant of $A$ is the product of the pivots in any echelon form $U$ of $A$, multiplied by $(-1)^r$, Where $r$ is

► the number of rows of $A$
► The number of row interchanges made during row reduction from $A$ to $U$
► The number of rows of $U$
► The number of row interchanges made during row reduction $U$ to $A$

Ref: If there are $r$ interchanges, then $\det(A) = (-1)^r \det(U)$

Question No: 10  ( Marks: 1 ) - Please choose one
If $A$ is invertible, then $\det(A) = \det(A^{-1}) = 1$.

► True
► False

Question No: 11  ( Marks: 1 ) - Please choose one
$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$  

A square matrix is lower triangular if and only if

$\begin{cases} a_{ij} = 0 & \text{for } i > j \\ i > j \\ i < j \end{cases}$
Question No: 12 (Marks: 1) - Please choose one
The product of upper triangular matrices is

- lower triangular matrix
- upper triangular matrix
- Diagonal matrix

Question No: 13 (Marks: 1) - Please choose one
The matrix multiplication is associative
- True Page no : 17
- False

Question No: 14 (Marks: 1) - Please choose one
We can add the matrices of ______________.
- same order Page no : 13
- same number of columns.
- same number of rows
- different order

Question No: 15 (Marks: 1) - Please choose one
By solving system of equations with iterative method, we stop the process when the entries in two successive iterations are ________.
- Repeat
- Large difference
- Different
- Same Page no : 196

Ref: Stop the process when the entries in two successive iterations are the same.

Question No: 16 (Marks: 1) - Please choose one
Jacobi’s Method is ____________ converges to solution than Gauss Siedal Method.
- slow Page no : 201
- fast
- better

Question No: 17 (Marks: 1) - Please choose one
A system of linear equations is said to be homogeneous if it can be written in the form ______________.
Question No: 18 (Marks: 1) - Please choose one
The row reduction algorithm applies only to augmented matrices for a linear system.

► True Page no : 43
► False Page no : 75

Question No: 19 (Marks: 1) - Please choose one
Whenever a system has no free variable, the solution set contains many solutions.

► True Page no : 75
► False Page no : 75

Question No: 20 (Marks: 1) - Please choose one
Which of the following is not a linear equation?

\[ x_1 + 4x_2 + 1 = x_3 \]

►

\[ x_1 = 1 \]

►

\[ x_1 + 4x_2 - \sqrt{2}x_3 = \sqrt{4} \]

►

\[ x_1 + 4x_1x_2 - \sqrt{2}x_3 = \sqrt{4} \]

Page no : 24

Question No: 1 (Marks: 1) - Please choose one
If A is a \(2 \times 2\) matrix, the area of the parallelogram determined by the columns of A is

► \(A\)

► \(\text{det } A\) Page no : 240

► \(\text{adj } A\)

Question No: 2 (Marks: 1) - Please choose one
Cramer’s rule leads easily to a general formula for

► The inverse of an \(n \times n\) matrix A Page no : 237
- the adjugate of an $n \times n$ matrix A
- the determinant of an $n \times n$ matrix A

**Question No: 3  (Marks: 1) - Please choose one**
The transpose of an lower triangular matrix is

- lower triangular matrix
- **Upper triangular matrix**
- diagonal matrix

**Question No: 4  (Marks: 1) - Please choose one**
The transpose of an upper triangular matrix is

- **Lower triangular matrix**
- upper triangular matrix
- diagonal matrix

**Question No: 5  (Marks: 1) - Please choose one**
Let $A$ be a square matrix of order $3 \times 3$ with $\det(A) = 21$, then $\det(2A) =$

- $168$
- $186$
- $21$
- $126$

**Question No: 7  (Marks: 1) - Please choose one**
Let $A$ be an $m \times n$ matrix. If for each $b$ in $\mathbb{R}^m$ the equation $Ax=b$ has a solution then

- A has pivot position in only one row  *(may be this option is true)*
- Columns of $A$ span $\mathbb{R}^m$
- Rows of $A$ span $\mathbb{R}^m$
Question No: 8 (Marks: 1) - Please choose one

Reduced echelon form of the matrix is

\[
\begin{bmatrix}
1 & 2 & 3 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

►

\[
\begin{bmatrix}
1 & 0 & 3 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

►

\[
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & 2 \\
1 & 0 & 0 \\
0 & 1 & 1 \\
\end{bmatrix}
\]

Question No: 6 (Marks: 1) - Please choose one

Col A is all of \( m \) if and only if

► the equation \( Ax = 0 \) has a solution for each \( b \) in \( m \)

► the equation \( Ax = b \) has a solution for each \( b \) in \( m \)

► the equation \( Ax = b \) has a solution for a fixed \( b \) in \( m \).

Question No: 7 (Marks: 1) - Please choose one

\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_1 \\
B_2 \\
\end{bmatrix}
\]

If and , then the partitions of A and B

► are not conformable for block multiplication

► are conformable for AB block multiplication

► are not conformable for BA block multiplication
Question No: 8  (Marks: 1) - Please choose one
Two vectors are linearly dependent if and only if they lie

- on a line parallel to x-axis
- **On a line through origin**  Page no : 87
- on a line parallel to y-axis

Question No: 9  (Marks: 1) - Please choose one
The equation \( x = p + t \, v \) describes a line

- through v parallel to p
- **through p parallel to v**  repeat
- through origin parallel to p

Question No: 10  (Marks: 1) - Please choose one
Let A be an \( m \times n \) matrix. If for each \( b \) in \( \mathbb{R}^m \) the equation \( Ax = b \) has a solution then

- A has pivot position in only one row  repeat
- Columns of A span \( \mathbb{R}^m \)
- Rows of A span \( \mathbb{R}^m \)

Question No: 11  (Marks: 1) - Please choose one
Given the system

\[
\begin{align*}
-x_1 + 2x_2 + x_3 &= 8 \\
2x_2 - 7x_3 &= 0 \\
-4x_1 + 3x_2 + 9x_3 &= -6
\end{align*}
\]

the augmented matrix for the system is

\[
\begin{bmatrix}
1 & -2 & 1 \\
0 & 2 & -7 \\
-4 & 3 & 9
\end{bmatrix}
\]
Consider the linear transformation $T$ such that $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -7 & 8 \\ -4 & 3 & 9 & -6 \end{bmatrix}$ is the matrix of linear transformation then $T$ is $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ is
Question No: 13  (Marks: 1) - Please choose one

\[
\begin{vmatrix}
  a & b & c \\
  d & e & f \\
  g & h & i \\
\end{vmatrix}
  = 5
\]

If \( \begin{vmatrix}
  a & b & c \\
  d & e & f \\
  g & h & i \\
\end{vmatrix} = 5 \) then \( \begin{vmatrix}
  3d & 3e & 3f \\
  g & h & i \\
\end{vmatrix} \) will be

15  Page no : 220
45
135
60

Ref:

- If one row of \( A \) is multiplied by \( k \) to produce \( B \), then \( \det B = k \cdot \det A \).

Question No: 14  (Marks: 1) - Please choose one

For an \( n \times n \) matrix \( (A^t)^t = \)

\( A^t \)

\( A \)  Page no : 18

\( A^{-1} \)

\( (A^{-1})^{-1} \)

Ref:

- The transpose of the transpose of a matrix is the matrix itself.
Question No: 15 (Marks: 1) - Please choose one
Each Linear Transformation $T$ from $\mathbb{R}^n$ to $\mathbb{R}^m$ is equivalent to multiplication by a matrix $A$ of order

- $m \times n$
- $n \times m$
- $n \times n$
- $m \times m$

Question No: 16 (Marks: 1) - Please choose one

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$$

Reduced echelon form of the matrix is

- $$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$
- $$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$
- $$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$
- $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Question No: 17 (Marks: 2)
Find vector and parametric equations of the plane that passes through the origin of $\mathbb{R}^3$ and is parallel to the vectors $v_1 = (1, 2, 5)$ and $v_2 = (5, 0, 4)$.

**Solution:**

$$x = t_1 v_1 + t_2 v_2$$

$X = (1, 2, 5).t_1 + (5, 0, 4).t_2$  This is vector equation of the plane.
These are parametric equations of the plane.

\[ X = x_0 + a_1 t_1 + a_2 t_2 \]
\[ Y = y_0 + b_1 t_1 + b_2 t_2 \]
\[ Z = z_0 + c_1 t_1 + c_2 t_2 \]

These are parametric equations of the plane.

**Question No: 22 (Marks: 3)**

Find that is invertible or not \( T(X_1, X_2) = T(6X_1 + 8X_2, 5X_1 - 8X_2) \)

**Solution:**

\[ T(6, 8) = T(5, -8) \]

\[
\begin{bmatrix}
6 & 8 \\
5 & -8
\end{bmatrix} = -48 + 40 = -8
\]

This is invertible.

**Question No: 23 (Marks: 3)**

Find the volume of parallelogram of the vertices \((1,2,4)\) \((2,4,-7)\) and \((-1,-3,20)\).

**Solution:**

Let

A\((1,2,4)\)
B\((2,4,-7)\)
C\((-1,-3,20)\)

Let's fix A
A(1,2,4)
B(2,4,-7)
C(-1,-3,20)

\[
\begin{bmatrix}
2 & -1 \\
4 & -2 \\
-7 & -4
\end{bmatrix}
= \begin{bmatrix} 1 \\ 2 \\ -11 \end{bmatrix}
\]

\[
\begin{bmatrix}
-1 & -1 \\
-3 & -23 \\
20 & -4
\end{bmatrix}
= \begin{bmatrix} -2 \\ -26 \\ 16 \end{bmatrix}
\]

Question No: 24  (Marks: 2)
Which of the following is true? If \( V \) is a vector space over the field \( F \).(justify your answer)

(a) \( \{ x + y \mid x, y \in V \} = V \)
(b) \( \{ x + y \mid x, y \in V \} = V \times V \)
(c) \( \{ \lambda v \mid v \in V, \lambda \in F \} = F \times V \)

Question No: 26  (Marks: 5)
Justify that \( A^2 = I \) if \( A = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} \), if and only \( M^2 = I \). justify your answer by portioned matrix of \( M \)
If a square idempotent matrix $A$ is non singular then show that $A$ is equal to the identity matrix $I$.

**Answer:**

Let $A = I$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Let's see whether it is non singular or not.

$$|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1 \neq 0$$

Hence it is obvious that $A$ is non singular while $A = I$
Question No: 23 (Marks: 3)

\[
\begin{vmatrix}
1 & 2 & 3 \\
-4 & 5 & 6 \\
7 & -8 & 9 \\
\end{vmatrix}
\]

Find

Answer:

**Example 3**

Find the determinant of the matrix \( A = \begin{bmatrix} 1 & 4 & 3 \\ 5 & 2 & 4 \\ 3 & 6 & 3 \end{bmatrix} \)

**Solution** Given \( A = \begin{bmatrix} 1 & 4 & 3 \\ 5 & 2 & 4 \\ 3 & 6 & 3 \end{bmatrix} \)

\[
|A| = \begin{vmatrix} 1 & 4 & 3 \\ 5 & 2 & 4 \\ 3 & 6 & 3 \end{vmatrix} = 1 \begin{vmatrix} 2 & 4 \\ 6 & 3 \end{vmatrix} - 4 \begin{vmatrix} 5 & 4 \\ 3 & 3 \end{vmatrix} + 3 \begin{vmatrix} 5 & 2 \\ 3 & 6 \end{vmatrix}
\]

\[= 1(2 \times 3 - 4 \times 6) - 4(5 \times 3 - 4 \times 3) + 3(5 \times 6 - 2 \times 3)
\]

\[= 1(-18) - 4(3) + 3(24)
\]

\[= -18 - 12 + 72
\]

\[= 42
\]
Question No: 25  (Marks: 5)
Show that \( A \) is invertible and find its inverse.

Answer:
\[
A = \begin{bmatrix}
I & 0 \\
A & I
\end{bmatrix}
\]
\[
det A = I^2 - 0
\]
\[
det A = I^2
\]
\[
I^2 = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]
\[
det A \neq 0
\]
so \( A \) is invertible
\[
A^{-1} = \begin{bmatrix}
I & 0 \\
A & I
\end{bmatrix}
\]
\[
A^{-1} = \frac{adjA}{det A} = \frac{\begin{bmatrix}
I & 0 \\
-A & I
\end{bmatrix}}{I^2} = \frac{1}{I} \begin{bmatrix}
I & 0 \\
-A & I
\end{bmatrix}
\]